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ABSTRACT

This project used an "intrinsic models" approach to develop a sample of microcomputer materials on graphing linear and quadratic equations and on recognition of general functional relationships. The "intrinsic models" approach uses motivating activities which are direct expressions of the underlying mathematics. The area of graphing was chosen because it is basic to scientific applications as well as to further work in pure mathematics. In addition, it is an area where tests indicate poor performance on the part of large numbers of students. The materials produced by this project deal with qualitative as well as quantitative aspects of graphs. Materials were developed in conjunction with student use in local public schools. Evaluation was largely formative, revolving around on-site observation of students, student interviews, and teacher interviews. Criterion-referenced test results from an informal (and incomplete) trial with the finished materials indicated that students learned well the concepts they had an opportunity to study. Descriptions of materials as presented in the instructor's manuals published with the programs are also provided. The first set of programs deals with graphing equations. The second set, recommended for high school mathematics/science or college science courses, helps students make meaningful interpretations of graphs of physical phenomena. (Author/JN)

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Prototype Microcomputer Courseware for Teaching High School Algebra

SED80-12449

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and
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Computer-based Education Research Laboratory
University of Illinois
Urbana, Illinois

Final Report
June 30, 1983

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Abstract

This project used an "intrinsic models" approach to develop a sample of materials on graphing linear and quadratic equations and on recognition of general functional relationships. The "intrinsic models" approach uses motivating activities which are direct expressions of the underlying mathematics.

The area of graphing was chosen because it is basic to scientific applications as well as to further work in pure mathematics. In addition, it is an area where tests indicate poor performance on the part of large numbers of students. The materials produced by this project deal with qualitative as well as quantitative aspects of graphs.

Materials were developed in conjunction with student use in the local public schools. Evaluation was largely formative, revolving around on-site observation of students, student interviews, and teacher interviews. Criterion-referenced test results from an informal (and incomplete) trial with the finished materials indicated that students learned well the concepts they had an opportunity to study.

The "Intrinsic Models" Approach

A common problem with the learning of mathematics is that students tend not to build a basic comfort and familiarity with concepts before moving on to material that depends on those concepts. When students are pushed ahead into manipulations of things that they are not totally comfortable with, it becomes easier to memorize a rule than to make sense of mathematical problems. Students then become "symbol-pushers" with no sense of what is actually happening in the mathematics and no sense of what might be a reasonable result (Erlwanger, 1975; National Assessment of Educational Progress, 1978). Once a student begins coping this way, the sense of the mathematics is lost and the foundation for future work becomes memorized algorithms. As more and more of these algorithms accumulate in the student's math bank, it becomes difficult to keep them all straight.

In response to this problem, there is a growing concern that applications should be a bigger part of the school mathematics program (Lesh, 1979; NACOME, 1975; Usiskin, 1979), not in the sense of the traditional "word problem", but in a more "real life" sense. Computers have offered potential for applications and simulations that were not previously possible in the classroom. In general, we have found the following characteristics very desirable in creating motivating and successful computer models for giving students experience with applying and exploring mathematical concepts. Activities that have these characteristics we refer to as "intrinsic models".

1. The student is given a working model to explore and manipulate.
2. The mathematics to be learned is intrinsic to the model.
3. The model provides a rich environment for exploration according to the student's own inclination and background.
4. Feedback is direct and meaningful, avoiding unnecessary verbiage and irrelevant gimmicks.
5. The mathematics is treated as inherently interesting, rather than hidden behind irrelevant themes.
6. Instructional activities are designed to be engaged in by students of widely varying background and ability in mathematics, but students find that the more math they apply, the better they do.
7. Rules are simple -- never more complex than the mathematics to be learned.
8. Feedback is diagnostic, so that students can tell at a glance what the error was and how it relates to a correct solution.

The Subject Matter

The area of graphing equations was chosen because it is basic to scientific applications as well as to further work in pure mathematics. In addition, it is an area where tests indicate poor performance on the part of large numbers of students (National Assessment of Educational Progress, 1978).

In the NAEP assessment of knowledge and skills, the level of student performance on algebra skills in general was viewed as "disheartening". Nearly all items included in the assessment would be considered part of a typical first year algebra course, and about $\frac{2}{3}$ of the 17-year-olds

reported that they had taken a year of Algebra I. However, items requiring use of algebra were answered correctly by only about 20-25% of the students. The study reports that graphing of lines receives a great deal of attention in both algebra and geometry classes, but student performance on such items was far lower than even the 20-25% correct for algebra items in general. To quote from the report,

"Given a coordinate set of axes and a ruler, 18% of the 17-year-olds correctly drew the graph of $y = 2x + 1$, and 7% correctly graphed the inequality $y \leq 2x + 1$. Four percent of the 17-year-olds wrote the correct equation of a line when they were given the x- and y- intercepts. Fifteen percent of the 17-year-olds named the slope of the line $2y = 5x - 8$, and 11% named the y-intercept of the same line."

With the increased attention to functions and graphing (Usiskin, 1979), these results are surprising, and would seem to indicate a need for improvement in the teaching of functions and graphs.

We hypothesize that the difficulty is due in part to students' lack of understanding of the functional relationships involved. Karplus (1979) notes students' difficulty in conceptualizing a functional relationship and suggests that mathematics instruction include extensive qualitative representations of functions by graphs. The materials developed by this project include activities relating physical phenomena to qualitative graphs (i.e. graphs that show the shape of the function without the units marked).

Implementation

Materials were developed in conjunction with student use in the Champaign, Illinois, public schools. Classroom testing of materials began in January 1981 with one microcomputer in a mathematics classroom. (See Appendix A for a description of the hardware used.)

All five of the programs described in this report (see "Materials Produced") were pilot tested and revised during the spring semester 1981. For this purpose, three microcomputers were locked to a long table at the back of one of the mathematics classrooms at Central High School. Several teachers used this room for classes when they were covering topics related to the materials available on the microcomputers. They seemed to find the inconvenience of trading classrooms worth it for the benefits.

Usage ranged from classes that used the room for two weeks to integrate the computer work with class work, to classes who used the room for two days to become familiar with the computers and encourage students to continue using them outside of class. Students often used the computers before or after school, during the noon hour, and at other free times during the day when a class was not using them.

Some teachers guided their students toward particular activities, while other teachers left the students completely on their own. Those teachers who did suggest particular activities tended to choose the "Graphing Equations" package as most closely related to the current classroom work.

A few teachers gently steered students away from the "Interpreting Graphs" package because it seemed less closely related to the equations they were teaching. This was not inconsistent with our observation that the qualitative aspects of graphs receive little attention in the mathematics curriculum.

Teachers were consistently concerned with seeing that all of the students in the class got a chance to use the computers. Typically, this was done by rotating a different set of students to the computers half-way through class and/or making sure that anyone who didn't get a chance one day had a chance the next day.

Usage was predominantly by groups of 2 or 3 students working together at each computer, but it was possible for students who wanted to work alone sometimes to do so. Teachers generally asked students to work in groups so that more students would be able to use the microcomputers. In general this worked well, since the group interaction encouraged students to talk about what they were doing and share ideas.

Some students, however, became deeply engrossed in working out complicated equations and seemed more intensely involved when working alone than would likely be possible in a group situation. These same students also analyzed other students' work through the replay option in the "Green Globes" records section (See program description).

It seemed that each of these modes of use (individual and small group) provided different important learning and exploration opportunities. Is is

interesting that the students we saw working in groups generally participated surprisingly equally. It was common to see students within the group frequently trading roles of advice giver and advice receiver.

Late in the Spring 1981 semester, four microcomputers were placed in a mathematics classroom at Centennial High School in Champaign, Illinois. Students in a geometry class used the materials for about three weeks as part of their classwork. These students and others were given a criterion referenced test before the microcomputer work began and at the end of the school year. Results are reported in the Evaluation section of this report.

Evaluation

Evaluation was largely formative. It revolved around on-site observation of students, student interviews, and teacher interviews. Such observations pinpointed any deficiencies that caused difficulty with the materials.

"Micro-assessment", or direct observation of students using the materials, has been basic to our work over the past years (Davis and Dugdale, 1976). An unobtrusive observer can easily see the same display the student is seeing, can observe which keys the student pushes, and can directly observe hesitations, changes, response times, etc. Data gathering is done on a scale of minutes, rather than weeks or months.

One important feature of the development and delivery system used is that materials could be changed quickly and easily, allowing the authors to revise lessons as soon as difficulties were observed. New versions could be tried within hours, a time scale radically different from that for piloting and revising text books, films, or even some computer curricula. The resulting materials are a product of extensive testing and revision.

In addition to the observational data, we used a criterion referenced test at one site to assess students' learning of some basic principles covered in the materials. For this purpose, four microcomputers were placed in a mathematics classroom in a school where the materials had not previously been used. Students in a geometry class used the materials for about three weeks as part of their classwork. Students taking algebra II from the same teacher were used as a comparison group. Both groups of students were given the criterion referenced test before the microcomputer work began and three weeks later at the end of the school year. The algebra I students were a year ahead of the geometry students (having taken geometry the previous year) and were studying in class the topics that our materials covered.

The algebra II students were expected not to use our materials. However, as sometimes happens when attractive materials are introduced into a classroom, it was difficult to limit use only to the students who were expected to use them.

The use of the materials by the comparison group was not the only problem. Due to some personnel difficulties within the school, it took longer than

expected to determine which teacher would be using the microcomputers, so the program was begun several weeks later than planned. Starting the program three weeks from the end of the school year prevented most treatment students from progressing as far as we would have liked.

From discussion with the teacher and students at the end of the year, it seems that the geometry students used our materials for about 2.5 to 3 hours each, with one exception which is noted below. The algebra II students were evidently exposed to the materials for an average of about 1.5 hours each, in somewhat less organized fashion than the geometry students.

From records kept by the students and the teacher, it appears that most of the students in the geometry class progressed through at least half of the materials on linear graphs. Few had progressed very far in quadratic graphs, and only one of the students completed all of the materials on quadratic graphs.¹

Figures 1 and 2 show the results for the criterion referenced test. Most of the test items required a constructed response, such as writing an equation for a given graph. The data presented here are based on the scores of all students who were present for both the pretest and posttest: 25 geometry students and 24 algebra II students.

¹This particular student became very involved and spent as much time as possible in the math classroom during non-class hours. He totalled about ten hours with the materials. He had responded correctly to only 2 of the 22 items on the pretest, but scored 100% on the posttest.

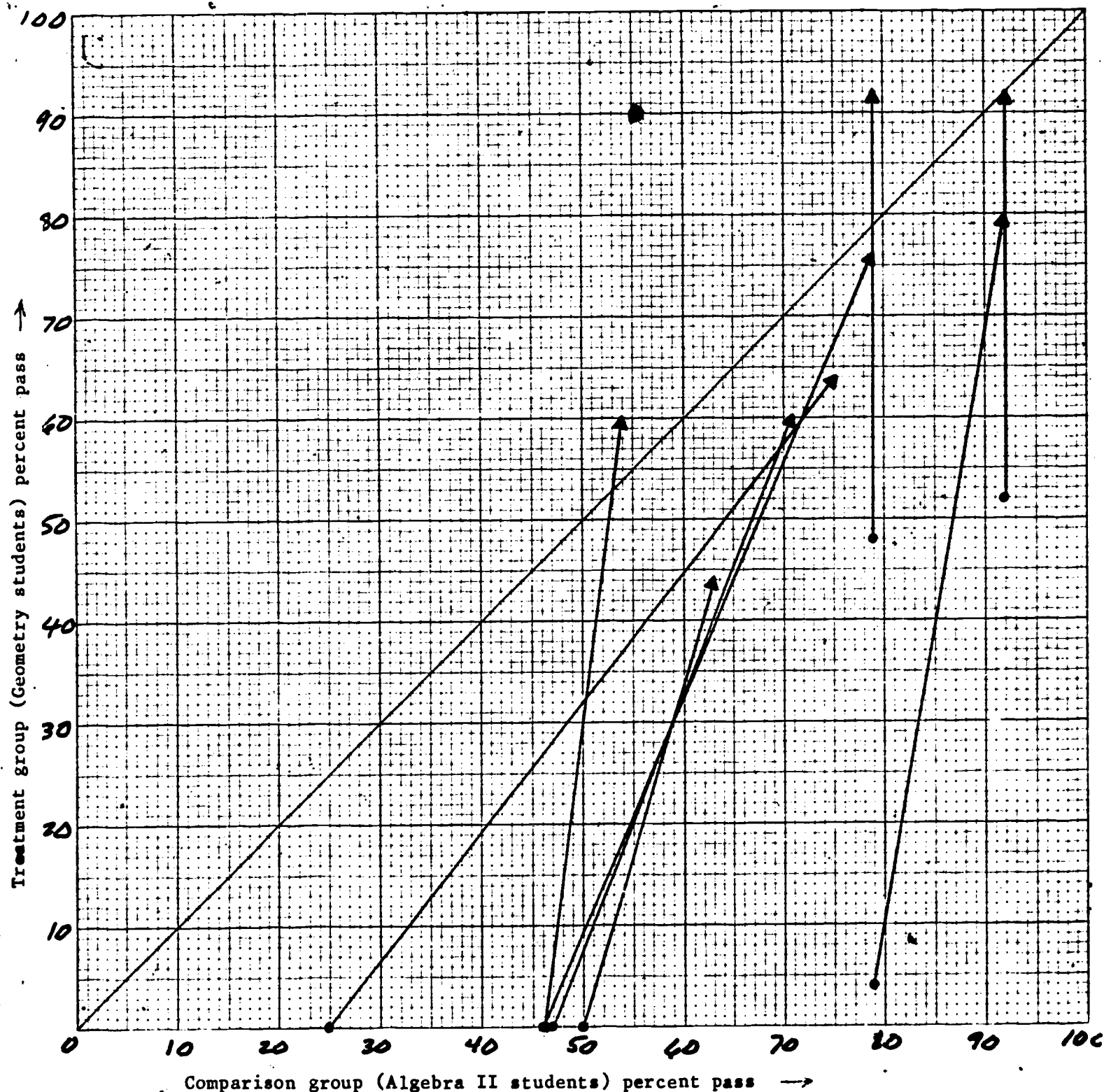


Fig. 1. Test results for items dealing with linear graphs. Each vector is associated with an item from the test. The tail of each vector is plotted at the point determined by the percentages of geometry students (the treatment group) and algebra II students (the comparison group) who passed the associated test item on the pretest. The head of the vector is plotted at the point determined by corresponding percentages from the posttest. Since the geometry class is on the ordinate, a steeply sloping vector indicates better relative performance by the geometry students. A slope less than 45 degrees indicates better relative performance by the algebra II class.

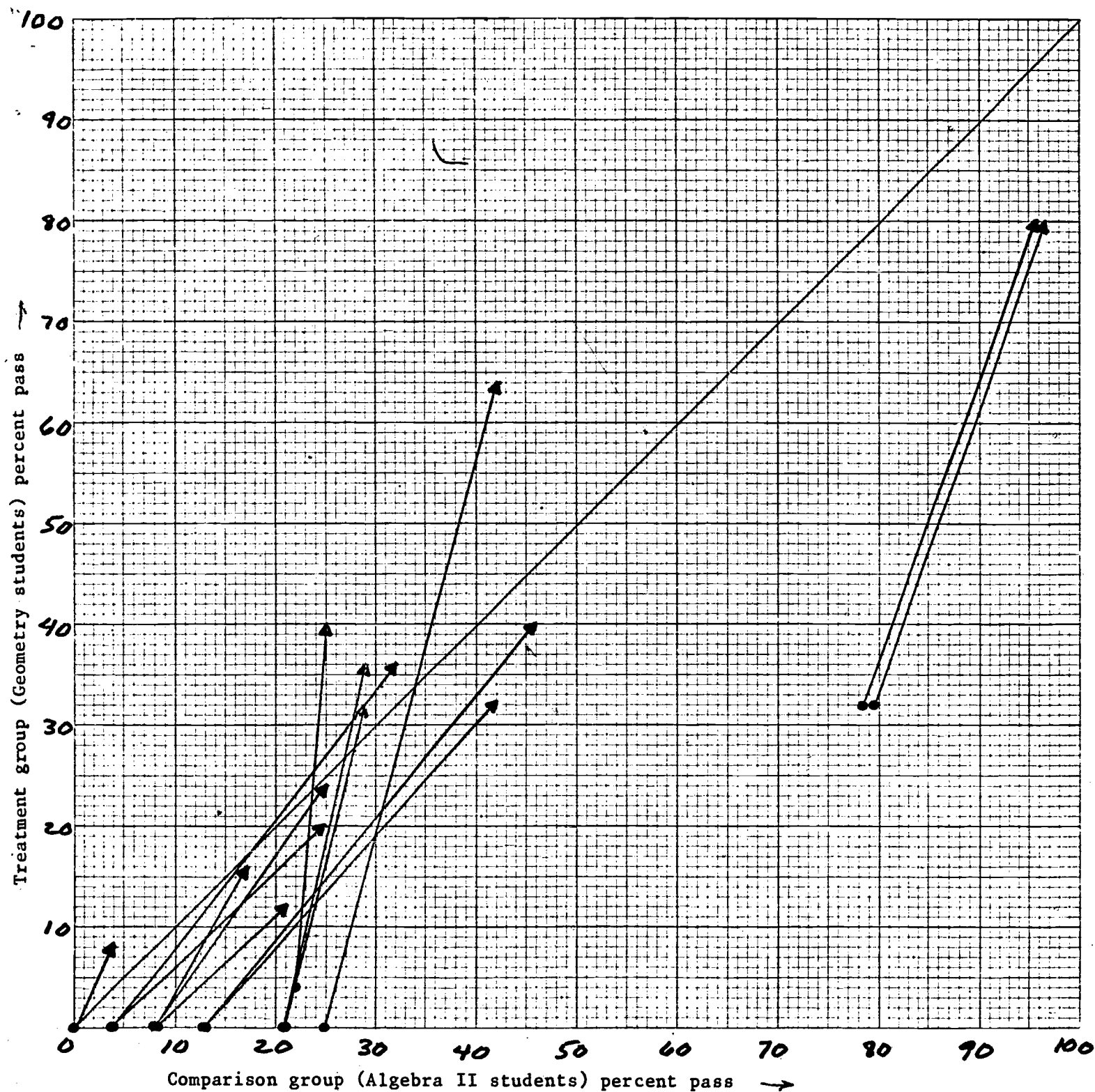


Fig. 2. Test results for items dealing with quadratic graphs.

In the figures all but two of the vectors have slopes greater than 45 degrees (indicating better relative performance by the geometry students), and many of them are quite steep. Overall, the greatest gains for the geometry students are on items dealing with linear equations, as could be expected since most students did not get very far into the materials dealing with quadratics.

The algebra II students' greater prior exposure to the topics is apparent from the pretest scores (shown by the tails of the vectors). Because the topics covered in our materials were a part of the algebra II curriculum, it could be expected that gains from pretest to posttest for those students in the last three weeks of the school year would reflect whatever preparation they had made for their final exam. It is interesting that the posttest performance on quadratic items (indicated by the heads of the vectors in fig. 2) is on the average fairly comparable for the two groups.

Materials Produced

The project produced two packages of materials, which are now available through Conduit, P.O. Box 388, Iowa City, IA 52244. The programs were converted to other microcomputers and revised before publication. The descriptions included here are from the instructor's manuals published with the programs. Both packages use color, but there is no problem using them on monochrome monitors as well. Several articles providing further discussion of these materials are listed in the references.

1. Graphing Equations

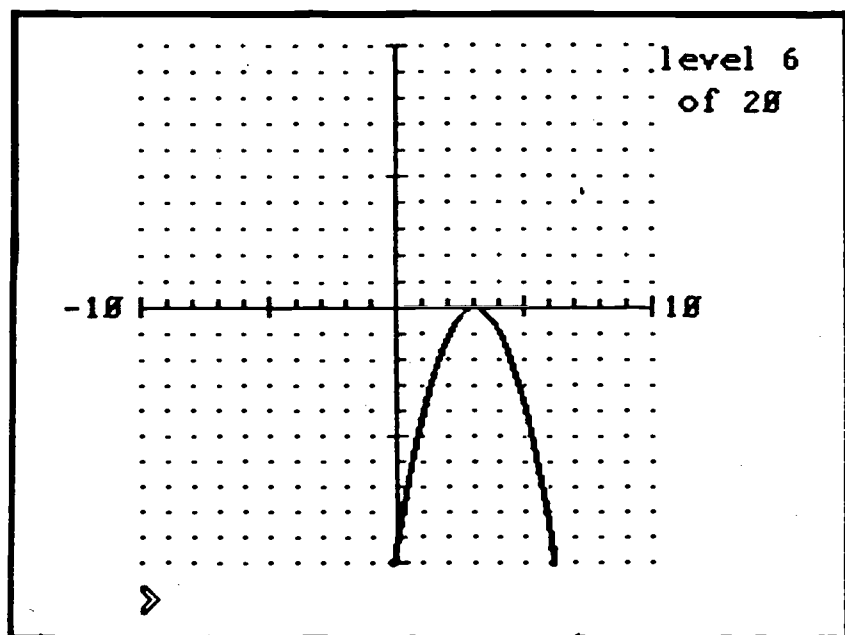
The programs in this package deal with graphing equations. This material is typically covered in high school algebra or in analytic geometry, but the programs are appropriate for any age student studying graphs of equations.

The programs "Linear and Quadratic Graphs" and "Tracker" address only linear and quadratic graphs. "Green Globes" permits students to use a wider variety of equations, including absolute value, square root, logarithmic, exponential, and trigonometric functions.

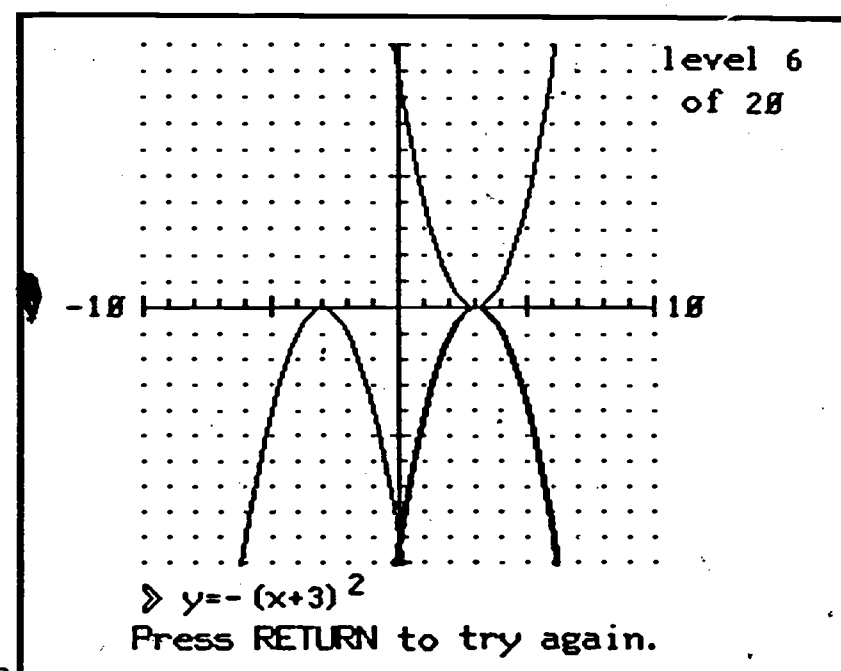
Linear and Quadratic Graphs

This program deals with graphing of straight lines, parabolas, circles, ellipses, and hyperbolas. The student's task is to write an appropriate equation for a given graph. The "target" graph is displayed in one color. Each equation the student writes is automatically graphed in a second color. The student can make successive tries and see how changes of the equation affect the graph, and how each try relates to the target graph and previous tries. This simple, graphic feedback for each incorrect equation helps the student diagnose his own errors.

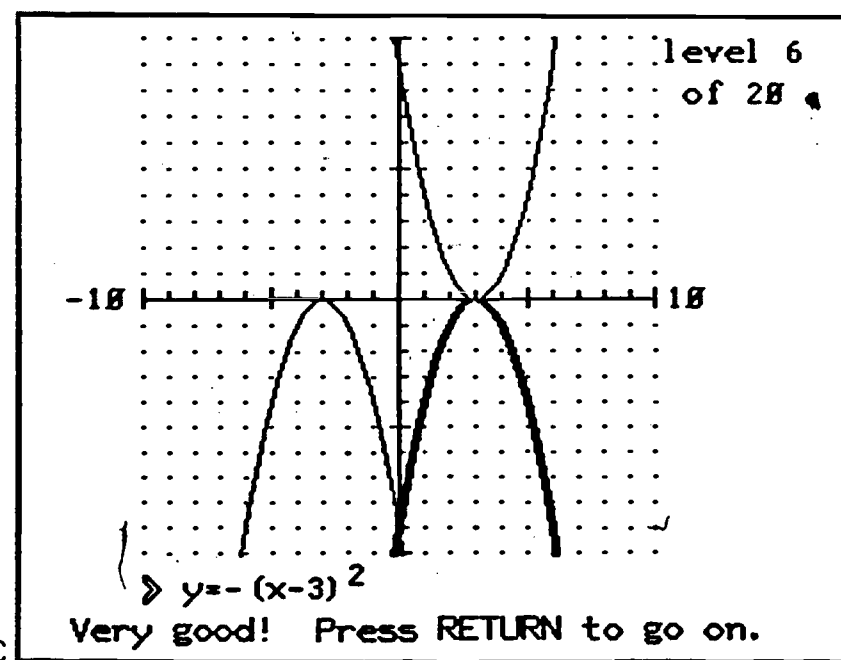
When the student's equation matches the target equation, his graph plots over the target graph. If the student's graph is only slightly different from the target graph, the program will call it "close enough" and proceed as for a correct response.



A



B



C

Fig. 3. Three successive screen displays. The first frame shows a graph for which the student must write an equation. In the second frame the student has made two incorrect responses. The equation entered in the third frame is correct.

Note: This program is based on "Exploring Simple Graphs", an experimental program designed by John Gilpin in 1979 at the University of Illinois.

The student can choose to work on straight lines, parabolas, circles, ellipses, or hyperbolas. For each type of graph, the program provides a sequence of difficulty levels, and the student has the following three options:

1. Begin with easy problems and work up to hard ones
2. Choose a higher starting level and work up from there
3. Practice problems of mixed difficulty

These options provide flexibility for students with different needs.

The sequence for each type of graph begins with the basic shape, then requires the student to manipulate this shape, with each level involving a different idea or more complexity. Nearly all levels are programmed to include several similar graphs, but there are a few early levels that include only one graph. For example, level 6 for parabolas includes the graphs $y=-(x-1)^2$, $y=-(x-2)^2$, $y=-(x-3)^2$, $y=-(x-4)^2$, and so on, but level 1 is always the graph $y=x^2$.

If a student matches a target graph on the first try, he advances to the next level. The student can also ask to see a correct equation for the graph. He must, however, answer an item correctly without asking for the answer in order to proceed to other levels.

In addition to selecting a particular type of graph, the student may also choose to practice a mixture of graphs. For example, if a student chooses to practice circles, ellipses, and hyperbolas, the program will present a sequence including all three types of graphs.

The goal of this program is not to have students memorize formulas, but rather for students to recognize basic shapes of graphs, to know in general how to translate graphs horizontally and vertically, and to perform other manipulations. For example, the student should learn to recognize a parabola as a $y=x^2$ shape, and also how to make the graph wider or narrower, how to reflect it through the x axis, and how to move it horizontally and vertically. These skills are generalizable to other types of graphs, and they enable the student to meaningfully produce a formula if one is needed.

The careful sequencing of levels available in this program enables some students to proceed without previous instruction, although the program is intended for students who are studying graphs in class.

Green Globes

The Novice Game. This program provides students an opportunity to use and broaden their knowledge of graphing equations. The computer displays coordinate axes with 13 green globes scattered randomly. The object of the game is to hit all of the globes with graphs specified by typing in equations. When a globe is hit, it explodes and disappears.

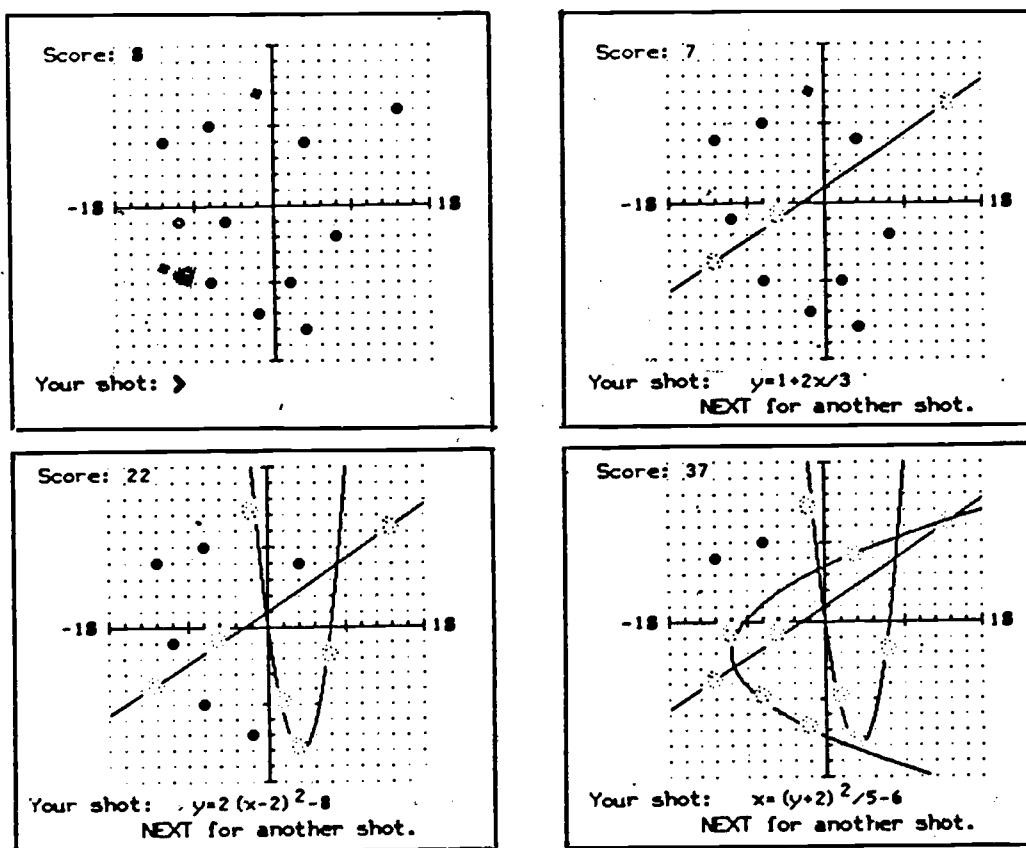


Fig. 4. Sequential displays from a game of Green Globs. The student types in equations, which are graphed by the computer. The green globes explode as they are hit by the graphs. Shown is the initial display of 13 globes, followed by the student's first three shots.

The scoring algorithm encourages students to hit as many globes as possible with each shot. For each shot, the first glob hit is worth one point, the second is worth two points, the third is worth four, and so on. For example, a five-glob shot will score $1+2+4+8+16$, for a total of 31 points.

Students can enter functions beginning with "y=" or "x=". For example, $y=x^2-3x+2$, $x=5-2y/3$, $x=(y+2)(y-1)^3$, and $y=3+x^5+5/x-(x-1)^3/(x^2-x)$ are all acceptable. The program also recognizes conics, for example, $x^2+5y^2=30$, $(x-4)^2/10-(y+2)^2/2=1$, $x^2=6$, and $xy=5$. Square root, absolute value, logarithmic, and exponential functions are accepted. Trig functions are excluded in the Novice Game because graphs like $y=10\sin(10x)$ could be used to hit all of the globs in one shot.

This activity is appropriate for students of widely varying backgrounds and abilities. It is clearly possible to hit all of the globs with linear functions (in fact with constant functions). However, the more mathematical knowledge the student applies, the more globs he or she is likely to hit per shot. Students of different levels are observed to develop various strategies and increase their skill with graphing. The game may be played by individual students or by two or more students working cooperatively.

When a student's shot misses the expected targets, the graphic feedback (display of the student's graph) gives diagnostic information needed for the student to debug his or her ideas about graphs. Perhaps the graph was too wide, or too steep, or upside down.

The top ten scores of all students who participate in the game are kept in a "hall of fame".

Green Globbs Top Ten Scores		
1.	145	Christy and Jeff
2.	138	Keiran
3.	97	Ted P. and Brian S.
4.	85	Leslie, Sal, and Shannon
5.	82	Martha
6.	76	Fred D., Barry, and Lori
7.	54	Arthur
8.	48	Michaela and Pat
9.	45	Steven, Cynthia, and Craig
10.	44	Dennis R. and Fred T.
Press BACK to return to the game, or choose a record number (from 1 to 10) to see a replay of it. >		

Fig. 5. The Green Globbs Hall of Fame. Students' names are displayed beside their record-making scores. From this display, students can choose to see a "replay" of any of the top ten games. Records are kept separately for Novice and Expert Games.

Students' names are displayed along with their record-making scores. A very important aspect of the activity is that all of the top ten games listed in the hall of fame are stored so that they can be viewed by other students who want to see what shots and strategies the top players have used. Students can "replay" the record-making games to gather tools that may be useful to them in future games.

The Expert Game. The Expert Game differs from the Novice Game in two ways:

1. In addition to the thirteen green globs, the coordinate grid contains five "shot absorbers", which stop any shots that touch them. (See fig. 6.) Students must try to avoid losing their graphs into the shot absorbers.
2. Students are allowed to enter trigonometric functions, as well as all of the equations that can be used in the Novice Game. For example, $y=\sin(x)$, $y=5\sec(2x)-3x+1$, $y=(x^2/5)(\cos(3x))$, and $x=1/(\tan(1/y^2))$ are all acceptable in the Expert Game.

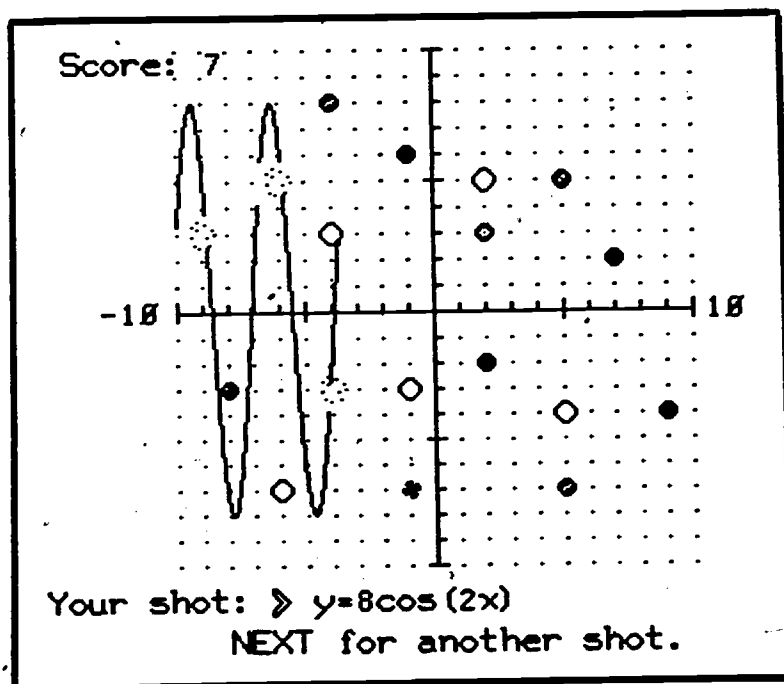


Fig. 6. In this display from the Expert Game, the student's graph was stopped when it hit a shot absorber.

In addition to the graphs typically studied in algebra, students can apply algebraic techniques to construct other interesting and useful graphs. For example, the graph $y = (x+5)(x-1)(x-7)$ can be constructed to cross the x axis at the points -5, 1, and 7. Using the coordinates of a target glob not on the x-axis, the student can find an appropriate factor to adjust the height of the graph to make it hit the desired globs. Figure 7 shows the use of another technique developed by a student during the first classroom use of this program.

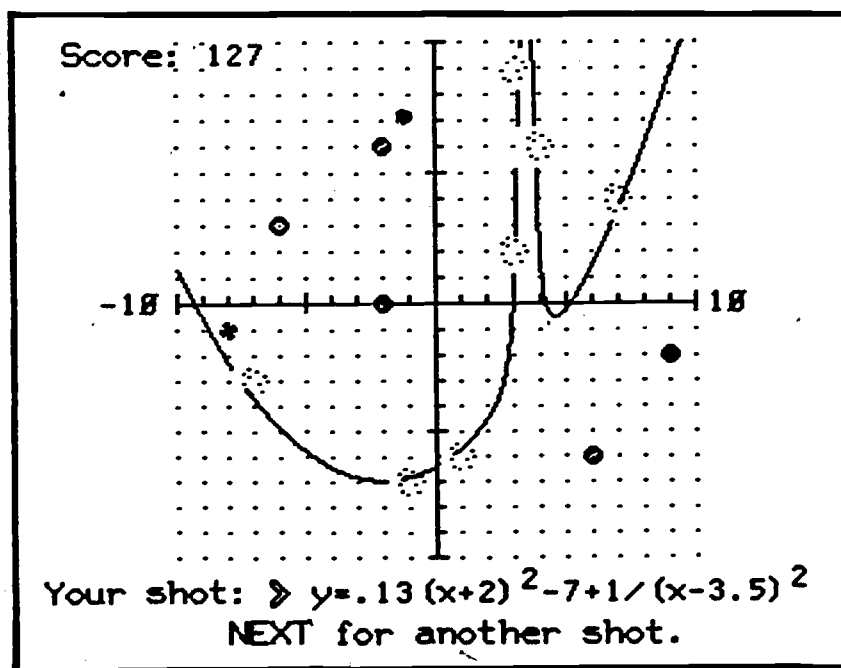


Fig. 7. The student has constructed a parabola to hit several green globs, but has then added $1/(x-3.5)^2$ to the function in order to make it jump up and capture three more globs near $x=3.5$.

Fig. 8 gives an example of what can be done in the Expert Game.

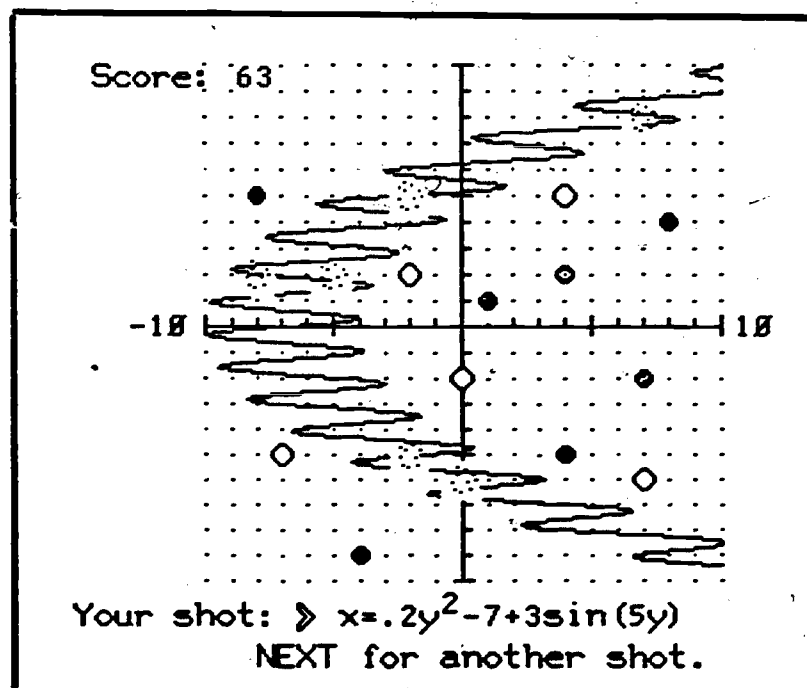


Fig. 8. A student in the Expert Game has constructed a parabola and then added $3\sin(5y)$ to the equation to make the graph cover a wider path, while still avoiding the shot absorbers.

Tracker

The object of this game is to locate linear and quadratic graphs that are "hidden" in the coordinate plane and determine their equations. The hidden graphs may be lines, parabolas, circles, ellipses, or hyperbolas, but there are only two types of graphs to find in any one game.

Students input two kinds of "shots" in this game: probes and trackers. Probes are used to find clues about the hidden graphs. After gathering enough clues to determine the equation of a graph, the student shoots a tracker along the graph.

A probe travels in a horizontal or vertical line, like $x=4$ or $y=-3$. The student launches a probe by writing its equation. As a probe goes across the screen, it marks each point where it crosses a hidden graph. (See fig. 9) Graphs of one type are marked with an 'X', and graphs of the other type are marked with an 'O'.

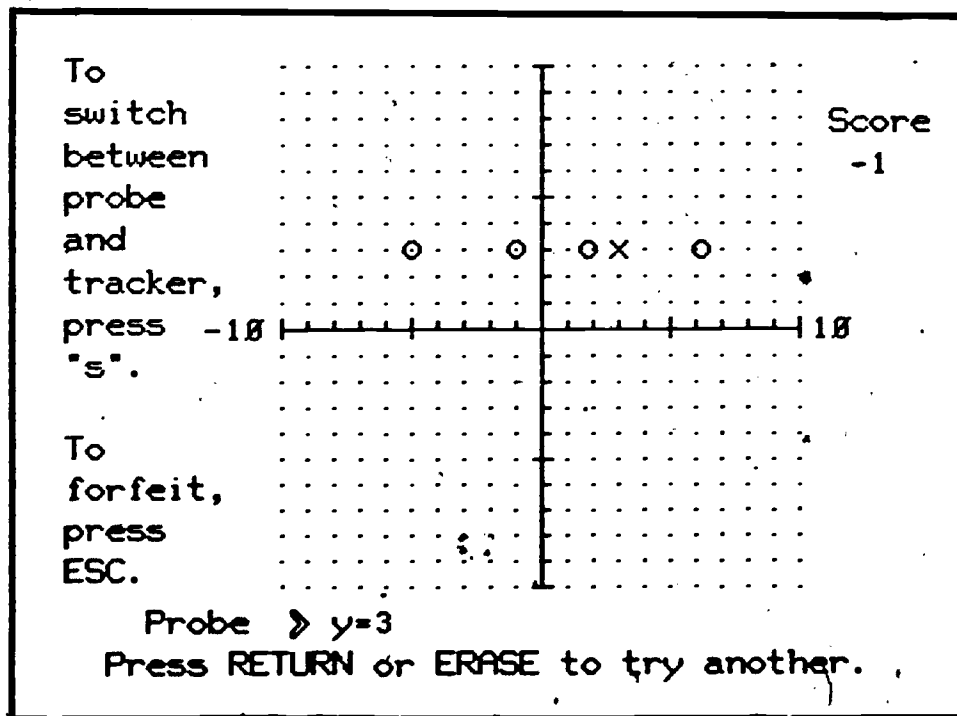


Fig. 9. A probe with the equation $y=3$ has intercepted hidden graphs at several points. X's mark one type of graph, while O's mark the other.

Depending on where a probe is placed, it may hit several hidden graphs or none at all. Probes mark only those graphs that have not yet been tracked. A hidden graph is never a horizontal or vertical line.

After using probes to locate a graph, the student sends a tracker along the graph by writing its equation. (See fig. 10.) Graphs that are very close to the hidden graph are accepted as "close enough".

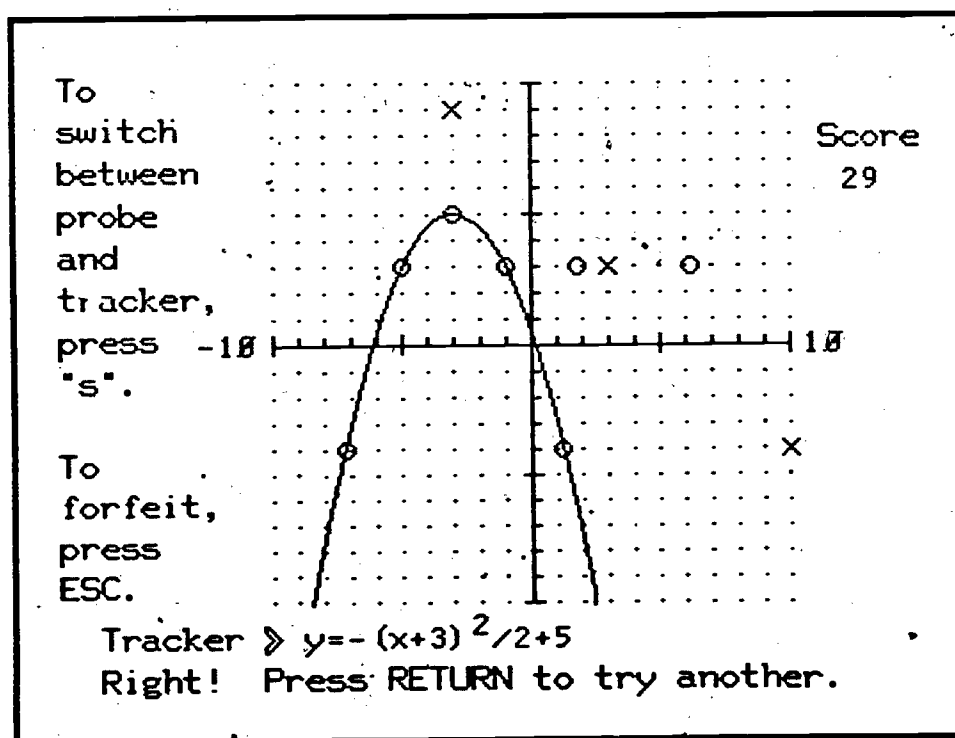


Fig. 10. Using several probes, the student has located a parabola. By writing the equation of the hidden graph, the student has sent a tracker along it. There are two more graphs left to track.

At any time, the student can switch between shooting probes (to find clues) and trackers (to trace a graph). There is also an option to forfeit the game and have the computer show any remaining hidden graphs.

The scoring algorithm encourages students to minimize the number of probes and trackers used. The 1st probe costs 1 point, the 2nd costs 2 points, the 3rd costs 3 points, and so on. Each wrong Tracker costs 10 points.

The points scored for a correct tracker depend on the type of graph. A successfully tracked straight line is worth 30 points, a parabola gets 35 points, a circle gets 40 points, an ellipse gets 45 points, and a hyperbola gets 50 points.

The top ten scores of all students who participate in the game are stored in the records. These record scores are kept separately for the Novice and Expert games. Scores must be greater than zero to qualify for the records, and games in which the student forfeited before finding all of the hidden graphs are not eligible for the records.

The Novice Game. In the novice game the student chooses two types of graphs, one to be marked with X's and the other with O's. There are always three graphs total hidden on the screen, two of one type and one of the other.

The Expert Game. In the Expert Game the computer chooses the two types of graphs, and the student must use probes to determine the types of graphs,

as well as their locations. The student chooses how many graphs are to be hidden (from three to six). Although higher scores are possible when there are more graphs to find, it can be very challenging to sort out more than three or four graphs on the screen.

Practice. In addition to the Novice and Expert Games, a simpler practice section is provided for students who wish to practice finding just one or two graphs. No score is kept in this section.

The experience of deciding which detected points could belong to a particular graph and which other points should be sought to confirm the presence of that graph furthers students' understanding of linear and quadratic relations.

Students need to use their probes efficiently by applying what they know about the graphs they are looking for. For example, if the sides of a parabola have been located, a reasonable strategy would be to find the vertex by shooting a probe half-way between them, along the axis of symmetry.

Students can plan similar strategies for locating key points for other types of graphs. For example, if a probe has crossed a circle, what additional probes are necessary to determine the coordinates of the center? Does it matter whether some of the circle is off the screen?

In deciding what additional probes would be most helpful, students also address questions like whether the slope of a line is more accurately computed from points that are close together or far apart.

The option to forfeit the game and have the computer show the graphs that the student did not find can be helpful in correcting misconceptions. For example, a student may assume that four points found by probes are sufficient to define two straight lines. After trying unsuccessfully to track the two lines, the student can ask to see the hidden graphs. It is sometimes a surprise to see that there is more than one way to put two lines through four points.

2. Interpreting Graphs

This package helps students make meaningful interpretations of graphs of physical phenomena. Because an understanding of graphs is especially important in scientific study, the programs in this package are recommended for students in high school mathematics courses or in introductory science courses at the high school or college level. Depending on the students' background, these programs could also be appropriate as low as the fifth grade level.

Relating Graphs to Events

This program introduces the use of graphs to describe physical phenomena. Following an introduction, there are twenty practice items in which the

computer graphs three functions in separate colors on the same axes. The axes have labels but do not have quantitative markings. A brief description of an event is written below the graph, and the student selects the graph whose shape is most appropriate for the given event. For users with monochrome monitors, the program provides a display with numbered graphs.

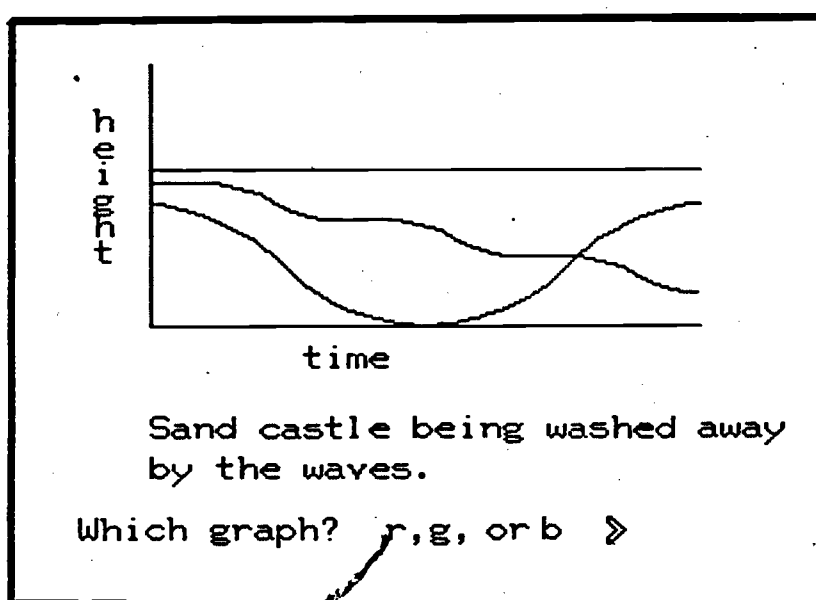


Fig. 11

Although the items do not appear in a fixed order, easier items are grouped toward the beginning of the program, with more challenging ones grouped toward the end.

When the student makes an incorrect choice, the computer responds with appropriate feedback about what the chosen graph describes.

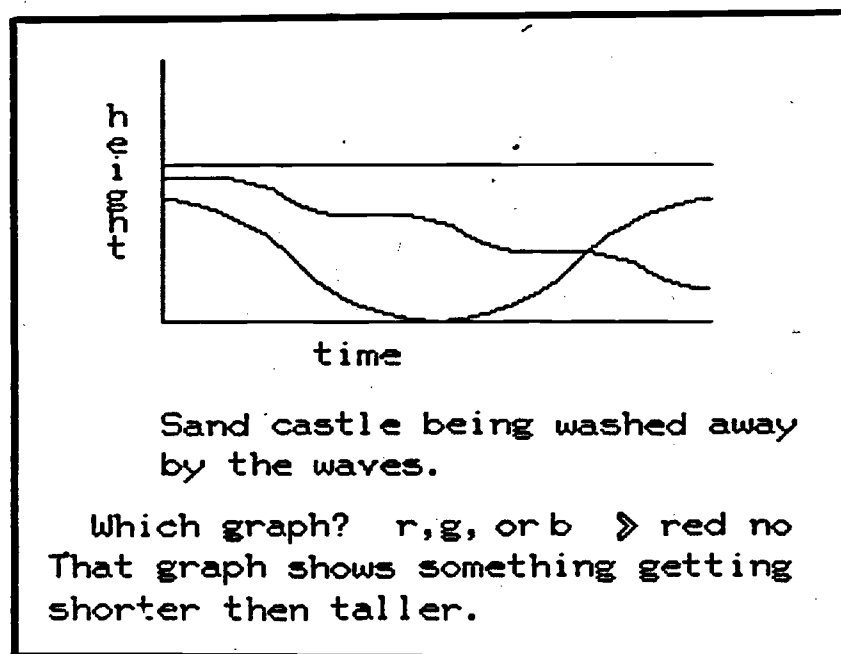


Fig. 12

The correct graph for each item is used as an incorrect choice for other items, so that students who choose the wrong graph for an item will also see that same graph in a context where it is the correct choice.

After the student has completed all twenty items, the program tells how many items were correct on the first try and offers a review of any items that were missed. Subsequent reviews of missed items are offered until the student has answered each item correctly on the first try. At that point, the program ends.

The emphasis of this program is on visualizing the general relationship between two variables, rather than on locating specific numerical values.

Hence, students are expected to develop an understanding of general functional relationships as expressed by graphs.

Often, students initially overlook the labels on the axes and simply choose a graph which seems to be a picture of the described event. For example, given a bicyclist going over a hill, a graph that looks like a hill might be an appropriate choice if the axes are labelled elevation vs. time, but it is an inappropriate choice when the axes are labelled speed vs. time. To emphasize that the labels on the axes are necessary for interpreting the graph, some items include choices that suggest a picture of the event, but that are, in fact, incorrect.

Other items are intended to raise ideas that might not occur to students. For example, a graph of the speed of a dog sleeping in the sun is not conceptually difficult, but it causes students to recognize that a constant zero graph is meaningful.

Escape

This game uses a rectangular coordinate grid as a city map. Bank robbers try to escape from the center of the city to the edge of the grid, while students try to trap them by setting blockades.

After the robbers' initial head start, they change direction only when they approach a blockade placed directly ahead of them. Their path is not displayed until after the game ends, so students must make blockade

decisions based on information graphed by the computer about the robbers' movements.

The student chooses one of three graphs (described below) for the computer to draw, and the information is not always complete -- for example, it might provide only the y-coordinate of the robbers' location. Students must decide what the graph tells them, what it does not tell them, and how to use this information to set blockades.

After each game, the student is offered the choice to play again using the same kind of graph or to choose a different graph. Each game has a difficulty level which adjusts to the student's performance. For higher levels the robbers take a bigger head start and move faster.

Graph #1: Direction Currently Headed. In this game the computer graphs the direction the bank robbers are moving (North, East, South, West).

Fig. 13 shows a game in progress. The robbers' path is not shown until the game is over, so the student must use the graph to decide where to put blockades.

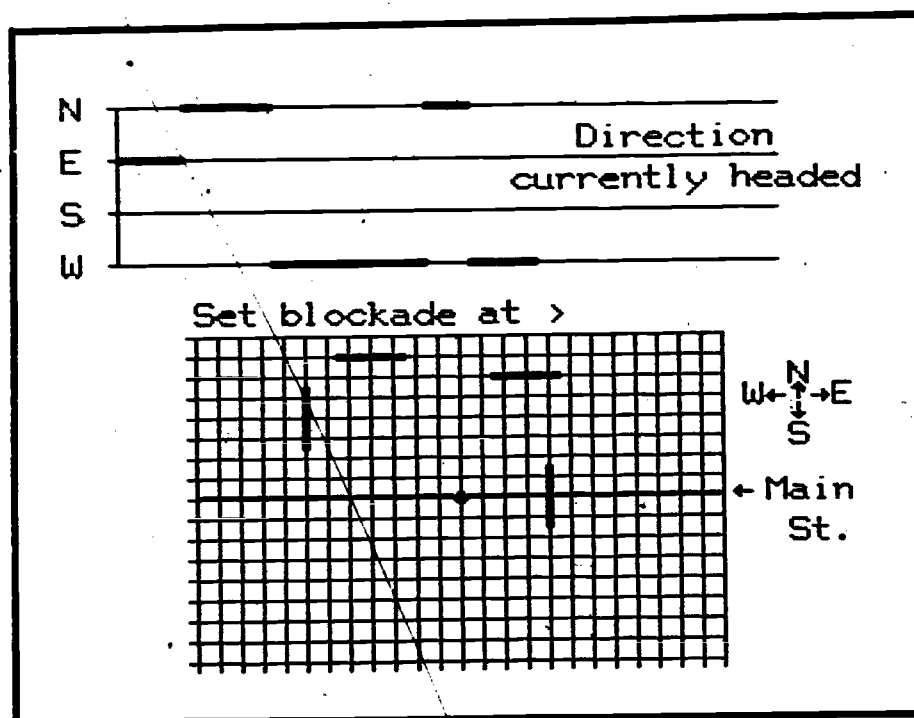


Fig. 13

Graph #2: Distance North of Main Street. In this game the computer graphs the robbers' distance north of Main Street. (Main Street is the x-axis of the city grid.) Fig. 14 shows a completed game. After the robbers have escaped, their path is shown so that the student can compare it with the graph and see if his interpretations were correct.

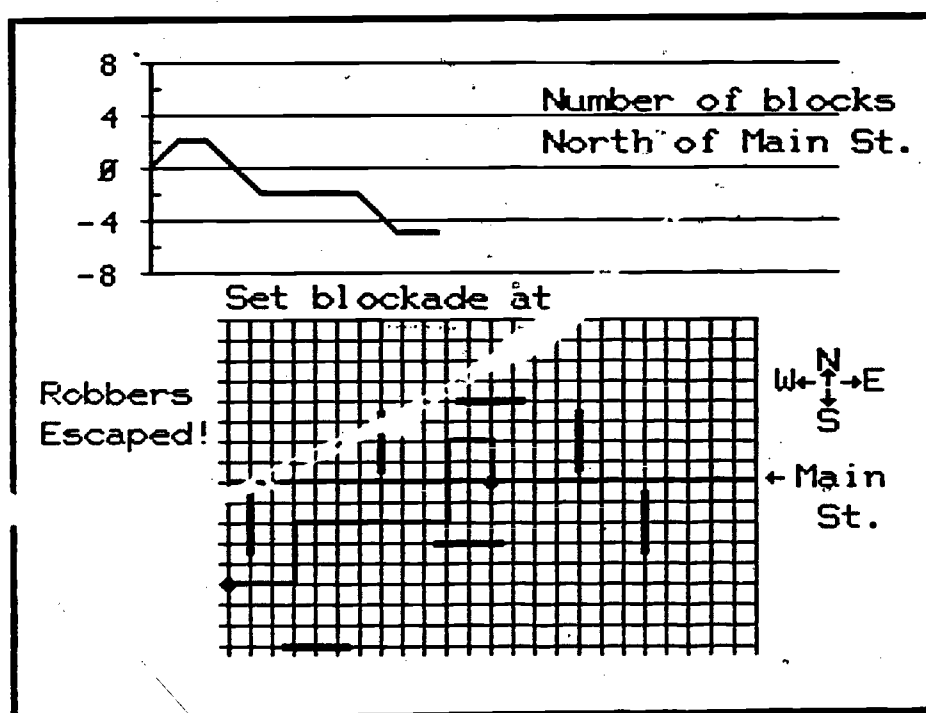


Fig. 14

Graph #3: Angle with East Main Street. In this game the computer graphs the angle between the robbers' location and East Main Street. (East Main Street is the positive part of the x-axis.) Fig. 15 shows a completed game. The student has caught the robbers with a blockade at $(-6, -7)$.

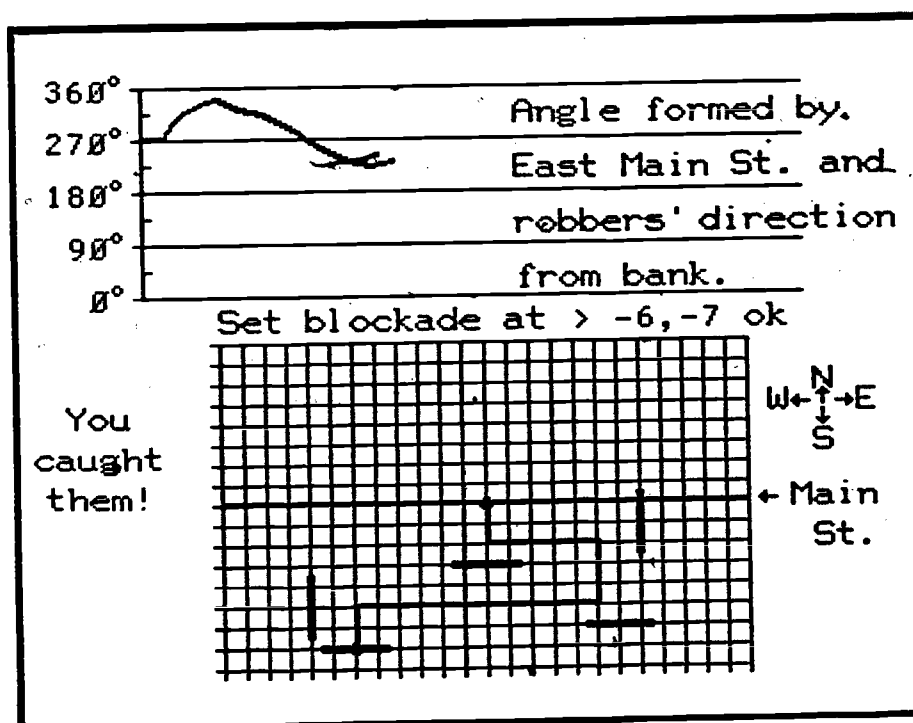


Fig. 15

Students playing Escape are doing more than just reading a graph. They are using the graph as a diagnostic tool, gaining important information when the robbers react (or fail to react) to blockades. Like many diagnostic tools, graphs in Escape may describe only one aspect of the situation. Part of the student's problem is to decide what to look for in the graph and how much accuracy is needed: sometimes the current value of the graph is what's important, sometimes the general range of recent values is all that's needed (and all that there is time for), sometimes the direction the graph is changing is what's important. The student must then decide what information is missing and how to place blockades to reduce the ambiguity. Games will usually involve successive attempts at a solution (especially at the intermediate and advanced levels), each attempt building on feedback from the previous attempts. Follow-up discussion suggestions are provided with the program.

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APPENDIX A

The Color Micro-PLATO System

The materials were designed for and tested on a color graphics microprocessor-based system recently developed at the Computer-based Education Research Laboratory (CERL) at the University of Illinois. The system uses an 8085 microprocessor, a SONY color television set, and an 8-inch floppy disk unit for program storage. The television set is modified by the addition of an external jack. A similar system (Stifle, et. al., 1979), using a plasma display screen instead of a color television, has been used for teaching chemistry at the University of Illinois.

Although this system can be connected by phone line to the PLATO network (Slottow, 1977; Smith and Sherwood, 1976) for use as an "intelligent" PLATO terminal, we used the system in stand-alone mode, completely independent of the PLATO system.

Two important features of the color display are:

1. Each of the 320 x 240 locations on the television screen is individually addressable in any of the 7 colors, regardless of the color of the adjacent locations.
2. The system keeps a separate bit map for each of the 3 primary colors. This allows us to move an object of one color around the screen without destroying the display behind it, a feature useful both in emphasizing parts of a display and in animation.

The programming language is Micro-TUTOR, a high level language developed by CERL for instructional use. The Micro-TUTOR language is designed for graphic displays and interactive lessons, e.g. lines are drawn just by specifying their endpoints, user-created alternative character sets are easily used, and powerful answer judging commands easily handle student input in many different formats. Micro-TUTOR is derived from the PLATO system's TUTOR, a language which has been evolving with widespread usage over a period of 15 years (Sherwood, 1977).

The system was originally used with 16K RAM to hold an instructional lesson. The RAM size does not limit the size of a lesson: if a lesson is larger than RAM, sections of it are brought in from the floppy disk as needed.

In spite of this, the initial development of our programs revealed a need for more memory in the microcomputer. Our programs required more than 16K RAM to hold all of the routines needed to accept, judge, evaluate and graph functions entered by the student. When only 16K was available, the earlier and later routines would take turns bumping each other out as they were read in, producing frequent disk accesses as the student entered several equations to be graphed. The delays had a marked effect on the pacing of the lessons. Within about a month of our initial classroom testing, all five of our microcomputers were upgraded from 16K RAM to 48K RAM, and the ROM was modified to increase the maximum number of units allowed in RAM at one time.

Instructional programs were written on-line to the PLATO system. Completed programs were converted by the on-line system into binaries which would run efficiently in the microprocessor and were copied to disk for classroom use. Development of materials on-line to the PLATO system gave us the advantage of using a mature, powerful system for development, together with the desirable features of delivering the materials on an economical microprocessor system. The PLATO system provided us several important features, including:

1. a high-level programming language developed specifically for instructional programs
2. editor support programs to allow efficient creation and revision of complex graphics/text displays
3. very fast creation of lesson binaries from source code (it takes just a few seconds for an entire large lesson), facilitating revisions and debugging

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